Solution problem -1

(a) $a_n = \dfrac{3+5n^2}{n+n^2} = \dfrac{(3+5n^2)/n^2}{(n+n^2)/n^2} = \dfrac{5+3/n^2}{1+1/n}$, so $a_n \to \dfrac{5+0}{1+0} = 5$ as $n \to \infty$. Converges

(b) $y = \left(1+\dfrac{2}{x}\right)^x \Rightarrow \ln y = x \ln\left(1+\dfrac{2}{x}\right)$, so

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(1+2/x)}{1/x} \overset{H}{=} \lim_{x\to\infty} \frac{\left(\dfrac{1}{1+2/x}\right)\left(-\dfrac{2}{x^2}\right)}{-1/x^2} = \lim_{x\to\infty} \frac{2}{1+2/x} = 2 \Rightarrow$$

$\lim\limits_{x\to\infty} \left(1+\dfrac{2}{x}\right)^x = \lim\limits_{x\to\infty} e^{\ln y} = e^2$, so by Theorem 3, $\lim\limits_{n\to\infty} \left(1+\dfrac{2}{n}\right)^n = e^2$. Converges

Solution problem -2

If $a_n = \dfrac{x^n}{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}$, then

$\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\dfrac{x^{n+1}}{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)(2n+1)} \cdot \dfrac{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}{x^n}\right| = \lim\limits_{n\to\infty} \dfrac{|x|}{2n+1} = 0 < 1$. Thus, by the Ratio Test, the series $\sum\limits_{n=1}^{\infty} \dfrac{x^n}{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}$ converges for *all* real x and we have $R = \infty$ and $I = (-\infty, \infty)$.

Solution problem -3

Forced Vibrations

See in 17.3, p. 1159, the equation of motion : $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$

And set c=0

The auxiliary equation for the homogenous equation has two imaginary roots $\pm j\omega$ so the solution is: $x_c(t) = c_1 \cos \omega t + c_2 \sin \omega t$

But the natural frequency of the system equals the

frequency of the external force, so try $x_p(t) = t(A\cos \omega t + B\sin \omega t)$. Then we need

$m(2\omega B - \omega^2 At)\cos \omega t - m(2\omega A + \omega^2 Bt)\sin \omega t + kAt\cos \omega t + kBt\sin \omega t = F_0\cos \omega t$ or $2m\omega B = F_0$ and

$-2m\omega A = 0$ (noting $-m\omega^2 A + kA = 0$ and $-m\omega^2 B + kB = 0$ since $\omega^2 = k/m$). Hence the general solution is

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t + \left[F_0 t/(2m\omega)\right]\sin \omega t$$

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1. (a) $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, 0 \leq x \leq 6, t \geq 0, x(0,t) = u(6,t) = 0, u(x,0) = 30, u(x,t) < M$. Let $u = X(x)T(t)$ plus boundary conditions $\Longrightarrow$ $u(x,t) = \sum B_m e^{-4m^2\pi^2 t/36} sin(m\pi x/6)$, hence $30 = \sum B_m sin(m\pi x/6)$ and $B_m = \frac{2}{6}\int_0^6 30 sin(m\pi x/6)dx = \frac{60}{m\pi}[1 - cos(m\pi)]$

   and

   (b) Now $u_x(0,t) = u_x(6,t) = 0$; the answer is obviously 30 degrees ...

2. Even function, so $b_n = 0$ and $a_n = \frac{2}{L}\int_0^L f(x)cos(n\pi x/L)dx \Longrightarrow$ $a_0 = 1/3$ and (partial integration) $a_n = (-1)^n \frac{4}{n^2\pi^2}$; $x = 1$ yields the required sum.

3. $F(\alpha) = \int_{-\infty}^{\infty} f(u)e^{-i\alpha u}du = \int_{-1}^{1} \frac{1}{2\epsilon} e^{-i\alpha u}du$ which equals $\frac{sin(\alpha)}{\epsilon\alpha}$. Within the ϵ interval $F(\alpha)$ becomes $\frac{sin(\alpha\epsilon)}{\epsilon\alpha}$ and the limit of this fraction goes to 1, as can be proven with the l'Hôpital theorem. This is the Dirac-δ function.